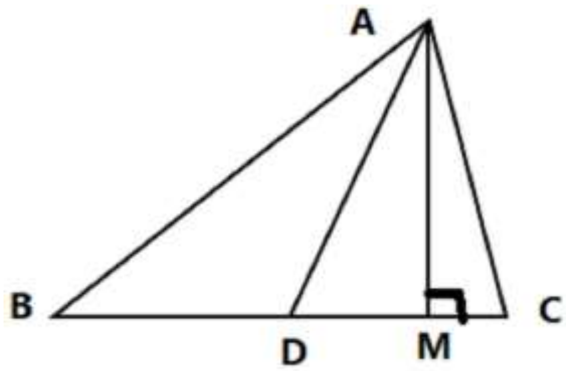
SOLUTION TO THE RIDER 01.02.2023

Claim: In triangle ABC, D is the mid point of BC,

then $AB^2 + AC^2 = 2(AD^2 + BD^2)$



ABC is a triangle, in which AD is a median on BC.
construction :- draw a line AM perpendicular to BC.
we have to prove : $AB^2 + AC^2 = 2(AD^2 + BD^2)$

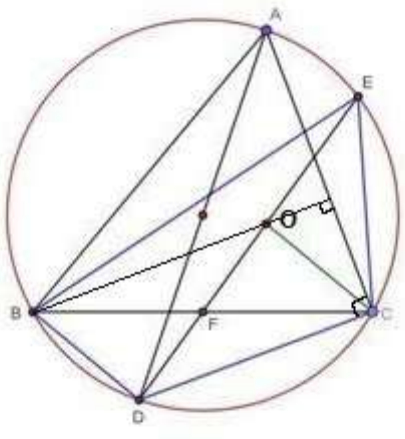
proof :-

case 1 :- when $\angle ADC = \angle ADB$, it means AD is perpendicular on BC and both angles are right angle e.g., 90°
then, from ΔADB,
according to Pythagoras theorem,
$AB^2 = AD^2 + BD^2$ ......(1)
from ΔADC ,
according to Pythagoras theorem,
$AC^2 = AD^2 + DC^2$ .... (2)
∵ AD is median.
so, BD = DC .... (3)
from equations (1) , (2) and (3),
$AB^2 + AC^2 = AD^2 + AD^2 + BD^2 + BD^2$
$AB^2 + AC^2 = 2(AD^2 + BD^2)$ [hence proved]

case 2 :- when $\angle ADB \neq \angle ADC$
Let us consider that, ADB is an obtuse angle.
from ΔABM,
from Pythagoras theorem,
$AB^2 = AM^2 + BM^2$
$AB^2 = AM^2 + (BD + DM)^2$
$AB^2 = AM^2 + BD^2 + DM^2 + 2BD.DM$ ...... (1)
from ΔACM,
according to Pythagoras theorem,
$AC^2 = AM^2 + CM^2$
$AC^2 = AM^2 + (DC - DM)^2$
$AC^2 = AM^2 + DC^2 + DM^2 - 2DC.DM$.......(2)
from equations (1) and (2),
$AB^2 + AC^2 = 2AM^2 + BD^2 + DC^2 + 2DM^2 + 2BD.DM - 2DC.DM$
$AB^2 + AC^2 = 2(AM^2 + DM^2) + BD^2 + DC^2 + 2(BD.DM - DC.DM)$ .... (3)
a/c to question, AD is median on BC.

so, BD = DC..... (4)
and from ADM,
according to Pythagoras theorem,
$AD^2 = AM^2 + DM^2$........ (5)
putting equation (4) and equation (5) in equation (3),
$AB^2 + AC^2 = 2AD^2 + 2BD^2 + 2(BD.DM - BD.DM)$
$AB^2 + AC^2 = 2(AD^2 + BD^2)$ [hence proved].

Now the given ques on:



From the figure BO||DC (Extended BO and DC are both perpendicular to AC.
OC||BD, (Extended CO and DB are perpendicular to AB)
Hence BOCD is a parallelogram.
BF = CF and DF = OF. (Diagonal bisect each other)
In triangle EBC, F is the mid point of BC, we can write (From the above claim)
$BE^2 + EC^2 = 2(EF^2 + BF^2)$ -------(1)
And in triangle DBC, F is the mid point of BC, we can write (From the above claim)
$CD^2 + DB^2 = 2(DF^2 + BF^2)$ --------(2)
By adding (1) and (2)
We can write $BE^2 + EC^2 + CD^2 + DB^2 = 2(EF^2 + BF^2 + DF^2 + BF^2)$
$\Rightarrow BE^2 + EC^2 + CD^2 + DB^2 = 2(EF^2 + DF^2 + 2BF^2)$
$\Rightarrow BE^2 + EC^2 + CD^2 + DB^2 = 2(EF^2 + DF^2 + 2BF \times CF)$ (As BF = CF)
$\Rightarrow BE^2 + EC^2 + CD^2 + DB^2 = 2(EF^2 + DF^2 + 2EF \times DF)$
(AS BF×CF=EF×DF) two chord of circle intersect at F
$\Rightarrow BE^2 + EC^2 + CD^2 + DB^2 = 2(EF + DF)^2$
$\Rightarrow BE^2 + EC^2 + CD^2 + DB^2 = 2DE^2$ (Proved)

DR. SHYAM SUNDAR AGRAWAL,
KOLKATTA.